MATHEMATICS

Chapter 6: Triangles

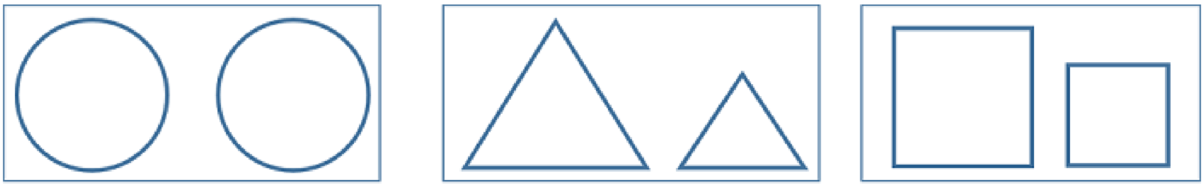
Triangles

1. **Congruent figures:**

   Two geometrical figures are called **congruent** if they superpose exactly on each other, that is, they are of the same shape and size.

2. **Similar figures:**

   Two figures are **similar**, if they are of the same shape but not necessarily of the same size.

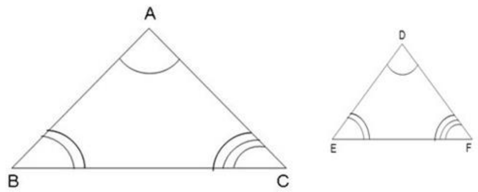


Group 1 Group 2 Group 3

3. All congruent figures are similar but the similar figures need not to be congruent.
4. Two **polygons** having the same number of sides are **similar** if
   i. their corresponding angles are equal and
   ii. their corresponding sides are in the same ratio (or proportion).

   Note: Same ratio of the corresponding sides means the **scale factor** for the polygons.
5. Important facts related to similar figures are:
   i. All circles are similar.
   ii. All squares are similar.
   iii. All equilateral triangles are similar.
   iv. The ratio of any two corresponding sides in two equiangular triangles is always same.
6. Two **triangles are similar** (~) if
   i. Their corresponding angles are equal.
   ii. Their corresponding sides are in same ratio.
7. If the angles in two triangles are:
   i. Different, the triangles are neither similar nor congruent.
   ii. Same, the triangles are similar.
   iii. Same and the corresponding sides are of the same size, the triangles are congruent.

In the given figure, $A \leftrightarrow D$, $B \leftrightarrow E$ and $C \leftrightarrow F$, which means triangles ABC and DEF are similar which is represented by $\Delta ABC \sim \Delta DEF$



8. If $\Delta\, ABC \sim \Delta\, PQR$, then
   i. $\Delta\, A = \Delta\, P$
   ii. $\Delta\, B = \Delta\, Q$
   iii. $\Delta\, C = \Delta\, R$
   iv. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$
9. **Equiangular triangles:**

   Two triangles are **equiangular** if their corresponding angles are equal. The ratio of any two corresponding sides in such triangles is always the same.
10. **Basic Proportionality Theorem** (Thales Theorem):

    If a line is drawn parallel to one side of a triangle to intersect other two sides in distinct points, the other two sides are divided in the same ratio.
11. **Converse of BPT:**

    If a line divides any two sides of a triangle in the same ratio then the line is parallel to the third side.
12. A line drawn through the mid-point of one side of a triangle which is parallel to another side bisects the third side. In other words, the line joining the mid-points of any two sides of a triangle is parallel to the third side.
13. **AAA (Angle-Angle-Angle) similarity criterion:**

    If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.
14. **AA (Angle-Angle) similarity criterion:**

    If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal.

    Thus, **AAA similarity criterion** changes to **AA similarity criterion** which can be stated as follows:

    If two angles of one triangle are respectively equal to two angles of other triangle, then the two triangles are similar.
15. **Converse of *AAA* similarity criterion:**

    If two triangles are similar, then their corresponding angles are equal.
16. **SSS (Side-Side-Side) similarity criterion:**

    If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

17. **Converse of SSS similarity criterion:**

If two triangles are similar, then their corresponding sides are in constant proportion.

18. **SAS (Side-Angle-Side) similarity criterion:**

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

19. **Converse of SAS similarity criterion:**

If two triangles are similar, then one of the angles of one triangle is equal to the corresponding angle of the other triangle and the sides including these angles are in constant proportion.

20. **RHS (Right angle-Hypotenuse-Side) criterion:**

If in two right triangles, hypotenuse and one side of one triangle are proportional to the hypotenuse and one side of another triangle, then the two triangles are similar. This criteria is referred as the RHS similarity criterion

21. **Pythagoras Theorem:**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Thus, in triangle ABC right angled at B, $AB^2 + BC^2 = AC^2$

22. **Converse of Pythagoras Theorem:**

If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

23. The **ratio of the areas of two similar triangles** is equal to the square of the ratio of their corresponding sides.

Thus, if $\Delta\, ABC \sim \Delta\, PQR$, then $\frac{ar\ \Delta\ ABC}{ar\ \Delta\ PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

Also, the ration of the areas of two similar triangles is equal to the ration of the squares of the corresponding medians.

24. Some important results of similarity are:

In an equilateral or an isosceles triangle, the altitude divides the base into two equal parts.

If a perpendicular is drawn from the vertex of the right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

The area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonal.

Sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

In an equilateral triangle, three times the square of one side is equal to four times the square of one of its altitudes.

25.**Triangle**

A triangle can be defined as a polygon which has three angles and three sides. The interior angles of a triangle sum up to 180 degrees and the exterior angles sum up to 360 degrees. Depending upon the angle and its length, a triangle can be categorized in the following types-

- Scalene Triangle – All the three sides of the triangle are of different measure
- Isosceles Triangle – Any two sides of the triangle are of equal length
- Equilateral Triangle – All the three sides of a triangle are equal and each angle measures 60 degrees
- Acute angled Triangle – All the angles are smaller than 90 degrees
- Right angle Triangle – Anyone of the three angles is equal to 90 degrees
- Obtuse-angled Triangle – One of the angles is greater than 90 degrees

26.Similarity Criteria of Triangles

To find whether the given two triangles are similar or not, it has four criteria. They are:

Side-Side- Side (SSS) Similarity Criterion – When the corresponding sides of any two triangles are in the same ratio, then their corresponding angles will be equal and the triangle will be considered as similar triangles.

Angle Angle Angle (AAA) Similarity Criterion – When the corresponding angles of any two triangles are equal, then their corresponding side will be in the same ratio and the triangles are considered to be similar.

Angle-Angle (AA) Similarity Criterion – When two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are considered as similar.

Side-Angle-Side (SAS) Similarity Criterion – When one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are said to be similar.

27.Proof of Pythagoras Theorem

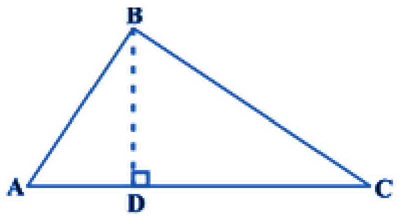
Statement: As per Pythagoras theorem, “In a right-angled triangle, the sum of squares of two sides of a right triangle is equal to the square of the hypotenuse of the triangle.”

Proof –

Consider the right triangle, right-angled at B.

Construction-

Draw $BD \perp AC$



Now, △ADB ~ △ABC

So, AD/AB = AB/AC

or AD. AC = AB^2 ....(i)

Also, △BDC ~ △ ABC

So, CD/BC = BC/AC

or, CD. AC = BC^2 ....(ii)

Adding (i) and (ii),

AD. AC + CD. AC = $AB^2 + BC^2$

AC (AD + DC) = $AB^2 + BC^2$

AC (AC) = $AB^2 + BC^2$

⇒ $AC^2 = AB^2 + BC^2$

Hence, proved.

28.Problems Related to Triangles

A girl having a height of 90 cm is walking away from a lamp-post's base at a speed of 1.2 m/s. Calculate the length of that girl's shadow after 4 seconds if the lamp is 3.6 m above the ground.

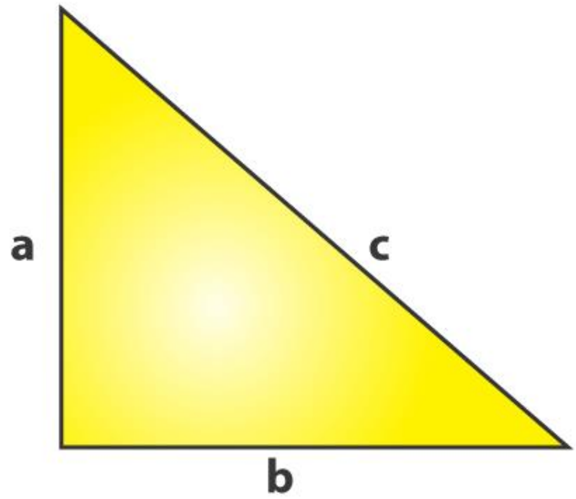
S and T are points on sides PR and QR of triangle PQR such that angle P = angle RTS. Now, prove that triangle RPQ and triangle RTS are similar.

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD

at F. Show that triangles ABE and CFB are similar.

29.Pythagoras Theorem Statement

Pythagoras theorem states that "In a right-angled triangle, the square of the hypotenuse side is equal to the sum of squares of the other two sides". The sides of this triangle have been named as Perpendicular, Base and Hypotenuse. Here, the hypotenuse is the longest side, as it is opposite to the angle 90°. The sides of a right triangle (say a, b and c) which have positive integer values, when squared, are put into an equation, also called a Pythagorean triple.



History

The theorem is named after a greek Mathematician called Pythagoras.

Pythagoras Theorem Formula

Consider the triangle given above:

Where "a" is the perpendicular,

"b" is the base,

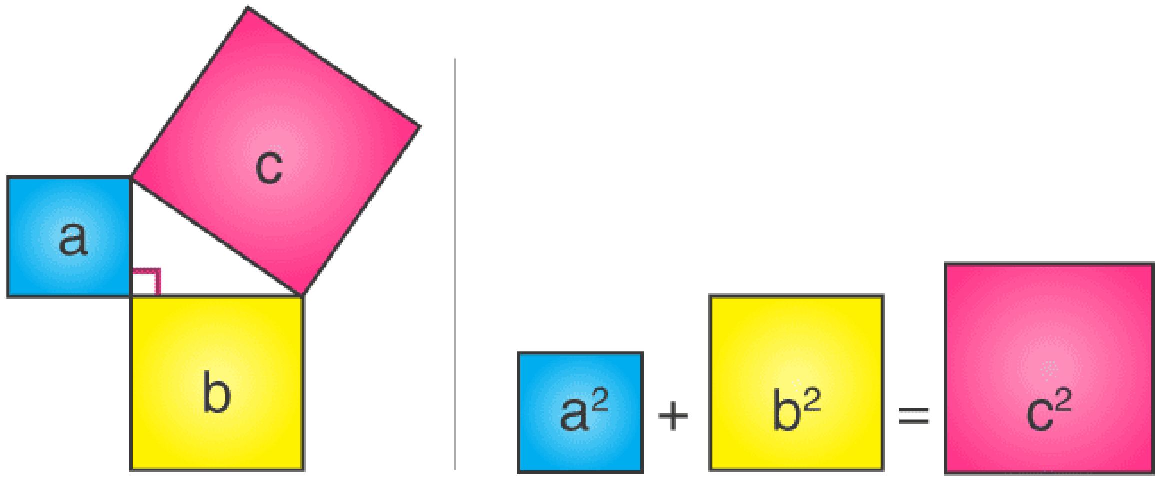
"c" is the hypotenuse.

According to the definition, the Pythagoras Theorem formula is given as:

$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$

$c^2 = a^2 + b^2$

The side opposite to the right angle (90°) is the longest side (known as Hypotenuse) because the side opposite to the greatest angle is the longest.



Consider three squares of sides a, b, c mounted on the three sides of a triangle having the same sides as shown.

By Pythagoras Theorem –

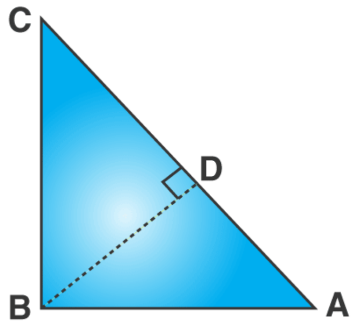
Area of square "a" + Area of square "b" = Area of square "c"

Pythagoras Theorem Proof

Given: A right-angled triangle ABC, right-angled at B.

To Prove- $AC^2 = AB^2 + BC^2$

Construction: Draw a perpendicular BD meeting AC at D.



Proof:

We know, △ADB ~ △ABC

Therefore,

$$\frac{AD}{AB} = \frac{AB}{AC}$$

(corresponding sides of similar triangles)

Or, $AB^2 = AD \times AC$ ……………………………………(1)

Also, △BDC ~△ABC

Therefore,

$$\frac{CD}{BC} = \frac{BC}{AC}$$

(corresponding sides of similar triangles)

Or, $BC^2 = CD \times AC$ ……………………………………(2)

Adding the equations (1) and (2) we get,

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$AB^2 + BC^2 = AC\ (AD + CD)$

Since, AD + CD = AC

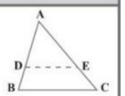
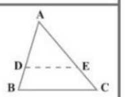
Therefore, $AC^2 = AB^2 + BC^2$

Hence, the Pythagorean theorem is proved.

MIND MAP : LEARNING MADE SIMPLE

Triangles

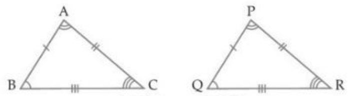
Theorems

Statement	Figure
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.	If, DE | | BC  then $\frac{AD}{DB} = \frac{AE}{EC}$
2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.	 If $\frac{AD}{DB} = \frac{AE}{EC}$ then, DE | | BC
3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)	If $\angle A = \angle D$, $\angle B = \angle E$ $\angle C = \angle F$ then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ $\Delta ABC \cong \Delta DEF$
4. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)	If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ then, $\angle A = \angle D$; $\angle B = \angle E$, $\angle C = \angle F$ $\Delta ABC \cong \Delta DEF$
5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)	If $\frac{AB}{DE} = \frac{AC}{DF}$ & $\angle A = \angle D$ then, $\Delta ABC \sim \Delta DEF$

Similarity

i) Corresponding angles are equal

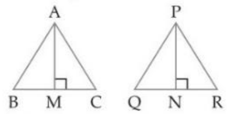
ii) Corresponding sides are in the same ratio



$\Delta ABC \sim \Delta PQR$

Area of Similar Triangles

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

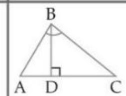


Here $\Delta ABC \sim \Delta PQR$

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Right angled triangle theorem

Pythagoras

Statement	Figure
1. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.	 In right ΔABC, $BD \perp AC$, then, $\Delta ADB \sim \Delta ABC$ $\Delta BDC \sim \Delta ABC$ $\Delta ADB \sim \Delta BDC$
2. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	In right ΔABC, $BC^2 = AB^2+AC^2$
3. In a triangle, if square of one side is equal to the sum of the squares of other two sides, then the angle opposite the first side is a right angle.	If $AC^2 = AB^2+BC^2$ then, $\angle B = 90°$

Important Questions

Multiple Choice questions-

1. If in triangles ABC and DEF, $\frac{AB}{EF} = \frac{AC}{DE}$, then they will be similar when

(a) ∠A = ∠D

(b) ∠A = ∠E

(c) ∠B = ∠E

(d) ∠C = ∠F

2.A square and a rhombus are always

(a) similar

(b) congruent

(c) similar but not congruent

(d) neither similar nor congruent

3. If ΔABC ~ ΔDEF and EF = $\frac{1}{3}$ BC, then ar(ΔABC):(ΔDEF) is

(a) 3 : 1.

(b) 1 : 3.

(c) 1 : 9.

(d) 9 : 1.

4. If a triangle and a parallelogram are on the same base and between same parallels, then what is the ratio of the area of the triangle to the area of parallelogram?

(a) 1 : 2

(b) 3 : 2

(c) 1 : 3

(d) 4 : 1

5. D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE || BC. Then, length of DE (in cm) is

(a) 2.5

(b) 3

(c) 5

(d) 6

6. Which geometric figures are always similar?

(a) Circles

(b) Circles and all regular polygons

(c) Circles and triangles

(d) Regular

7. $\Delta ABC \sim \Delta PQR$, $\angle B = 50°$ and $\angle C = 70°$ then $\angle P$ is equal to

(a) 50°

(b) 60°

(c) 40°

(d) 70°

8. In triangle DEF,GH is a line parallel to EF cutting DE in G and and DF in H. If DE = 16.5, DH = 5, HF = 6 then GE = ?

(a) 9

(b) 10

(c) 7.5

(d) 8

9. In a rectangle Length = 8 cm, Breadth = 6 cm. Then its diagonal = …

(a) 9 cm

(b) 14 cm

(c) 10 cm

(d) 12 cm

10. In triangle ABC ,DE || BC AD = 3 cm, DB = 8 cm AC = 22 cm. At what distance from A does the line DE cut AC?

(a) 6

(b) 4

(c) 10

(d) 5

Very Short Questions:

1. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

2. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm, and PB = 4 cm. Is AB || QR? Give reason.

3. If ΔABC ~ ΔQRP, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then find the length of PR.

4. If it is given that ΔABC ~ ΔPQR with $\frac{BC}{QR} = \frac{1}{3}$, then find $\frac{ar(\Delta PQR)}{ar(\Delta ABC)}$

5. ΔDEF ~ ΔABC, if DE : AB = 2 : 3 and ar(ΔDEF) is equal to 44 square units. Find the area (ΔABC).

6. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.

7. In triangles PQR and TSM, ∠P = 55°, ∠Q = 25°, ∠M = 100°, and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

8. If ABC and DEF are similar triangles such that ∠A = 47° and ∠E = 63°, then the measures of ∠C = 70°. Is it true? Give reason.

9. Let ΔABC ~ ΔDEF and their areas be respectively 64 cm^2 and 121 cm2. If EF = 15.4 cm, find BC.

10. ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.

Short Questions :

1. In Fig. 7.10, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.

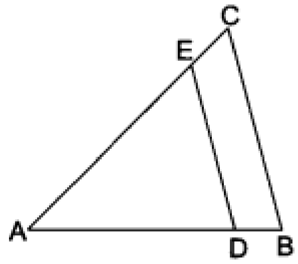


Fig. 7.10

2. E and F are points on the sides PQ and PR respectively of a ΔPQR. Show that EF ||QR if PQ = 1.28 cm, PR= 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

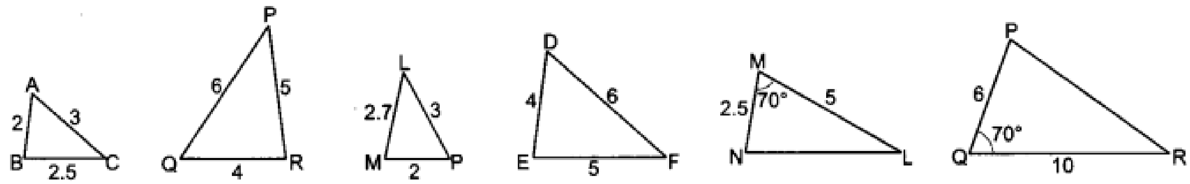
3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

4. In Fig. 7.13, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$

5. In Fig. 7.14, DE || OQ and DF || OR Show that EF || QR.

6. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

7. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.



8. In Fig. 7.17, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5cm. Find the value of DC.

9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

10. S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Long Questions :

1. Using Basic Proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

3. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

4. In Fig. 7.37, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

5. Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.

6. If the areas of two similar triangles are equal, prove that they are congruent.

7. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

8. In Fig. 7.41,0 is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

9. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see Fig. 7.42). Prove that $2AB^2 = 2AC^2 + BC^2$

10. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Case Study Questions:

1. Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.

i. Rahul tied the sticks at what angles to each other?

a. 30º
b. 60º
c. 90º
d. 60º

ii. Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite?

a. RHS
b. SAS
c. SSA
d. AAS

iii. Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio:

a. 2 : 3
b. 4 : 9
c. 81 : 16
d. 16 : 81

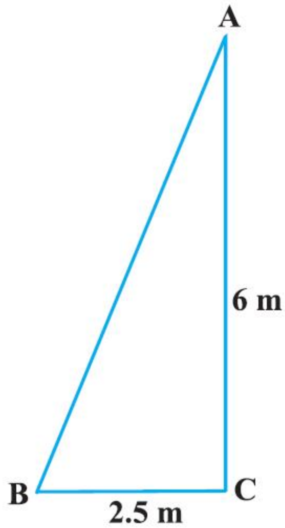
iv. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. This theorem is called.

a. Pythagoras theorem
b. Thales theorem
c. The converse of Thales theorem
d. The converse of Pythagoras theorem

v. What is the area of the kite, formed by two perpendicular sticks of length 6cm and 8cm?

a. 48 cm^2
b. 14 cm^2
c. 24 cm^2
d. 96 cm^2

2. There is some fire incident in the house. The fireman is trying to enter the house from the window as the main door is locked. The window is 6m above the ground. He places a ladder against the wall such that its foot is at a distance of 2.5m from the wall and its top reaches the window.



i. Here, ________ be the ladder and ________ be the wall with the window.

a. CA, AB
b. AB, AC
c. AC, BC
d. AB, BC

ii. We will apply Pythagoras Theorem to find length of the ladder. It is:

a. $AB^2 = BC^2 - CA^2$
b. $CA^2 = BC^2 + AB^2$
c. $BC^2 = AB^2 + CA^2$
d. $AB^2 = BC^2 + CA^2$

iii. The length of the ladder is ________.

a. 4.5m
b. 2.5m
c. 6.5m
d. 5.5m

iv. What would be the length of the ladder if it is placed 6m away from the wall and the window is 8m above the ground?

a. 12m
b. 10m
c. 14m
d. 8m

v. How far should the ladder be placed if the fireman gets a 9m long ladder?

a. 6.7m (approx.)
b. 7.7m (approx.)
c. 5.7m (approx.)
d. 4.7m (approx.)

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

Reason: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles.

Reason: All congruent triangles are similar but the similar triangles need not be congruent.

Answer Key-

Multiple Choice questions-

1. (b) ∠A = ∠E
2. (d) neither similar nor congruent
3. (c) 1 : 9.
4. (a) 1 : 2
5. (b) 3
6. (b) Circles and all regular polygons
7. (b) 60°
8. (a) 9
9. (c) 10cm
10. (a) 6

Very Short Answer :

1. Since the perimeters and two sides are proportional

∴ The third side is proportional to the corresponding third side.

i.e., The two triangles will be similar by SSS criterion.

2.

Yes, $\dfrac{PA}{AQ} = \dfrac{5}{12.5-5} = \dfrac{5}{7.5} = \dfrac{2}{3}$

$\dfrac{PB}{BR} = \dfrac{4}{6} = \dfrac{2}{3}$

Since $\dfrac{PA}{AQ} = \dfrac{PB}{BR} = \dfrac{2}{3}$

∴ $AB \parallel QR$

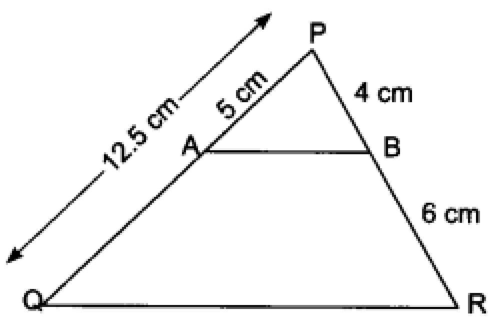


Fig. 7.4

3.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta QRP} = \frac{BC^2}{RP^2} \quad \Rightarrow \quad \frac{9}{4} = \frac{(15)^2}{RP^2}$$

$$\therefore \quad RP^2 = \frac{225 \times 4}{9} = \frac{900}{9} = 100 \Rightarrow RP = 10 \text{ cm}$$

4.

$$\frac{BC}{QR} = \frac{1}{3} \qquad \text{(Given)}$$

$$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{(QR)^2}{(BC)^2}$$

[$\because$ Ratio of area of similar triangles is equal to the ratio of square of its corresponding sides]

$$= \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9 : 1$$

5.

Since $\Delta DEF \sim \Delta ABC$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{(DE)^2}{(AB)^2}$$

[$\because$ Ratio of area of similar triangles is equal to the ratio of square of its corresponding sides]

$$\frac{44}{ar(\Delta ABC)} = \left(\frac{2}{3}\right)^2 \quad \Rightarrow \quad ar(\Delta ABC) = \frac{44 \times 9}{4}$$

So, $ar(\Delta ABC) = 99 \text{ cm}^2$

6. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 182$

$\therefore$ The given triangle is not a right triangle.

7. Şince, ∠R = 180° – (∠P + ∠Q)

= 180° – (55° + 25°) = 100° = ∠M

∠Q = ∠S = 25° (Given)

ΔQPR ~ ΔSTM

i.e., . ΔQPR is not similar to ΔTSM.

8. Since ΔABC ~ ΔDEF

$\therefore$ ∠A = ∠D = 47°

∠B = ∠E = 63°

$\therefore$ ∠C = 180° – (∠A + ∠B) = 180° – (47° + 63°) = 70°

∴ Given statement is true.

9.

We have, $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2}$ = (as $\Delta ABC \sim \Delta DEF$)

$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \frac{BC}{15.4} = \frac{8}{11}$

$\therefore BC = \frac{8}{11} \times 15.4 = 11.2$ cm

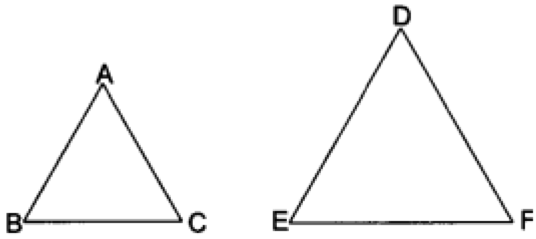


Fig. 7.5

10. ΔABC is right-angled at C.

∴ $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = AC^2 + AC^2$

[∵ AC = BC]

$\Rightarrow AB^2 = 2AC^2$

Short Answer :

1. In ΔABC, we have

DE || BC,

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4$

2.

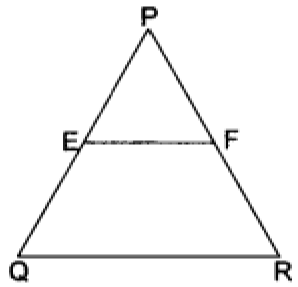


Fig. 7.11

We have, PQ = 1.28 cm, PR = 2.56 cm

PE = 0.18 cm, PF = 0.36 cm

Now, EQ = PQ-PE = 1.28 – 0.18 = 1.10 cm and

FR = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

and, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\quad\therefore\quad \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF || QR [By the converse of Basic Proportionality Theorem]

3. Let AB be a vertical pole of length 6m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

Now, in ΔABC and ΔDEF, we have

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the Sun)

$\therefore$ $\Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow$ $\frac{6}{h} = \frac{4}{28}$ (Let $DE = h$)

$\Rightarrow$ $\frac{6}{h} = \frac{1}{7}$ $\Rightarrow h = 42$

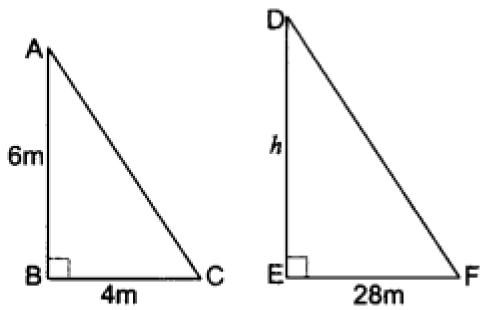


Fig. 7.12

h = 42 Hence, height of tower, DE = 42m

4.

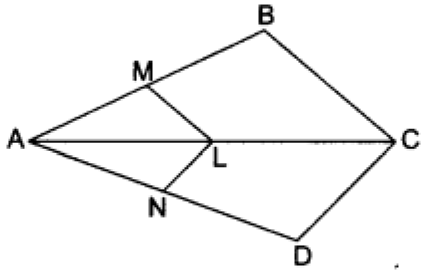


Fig. 7.13

Firstly, in ΔABC, we have

LM || CB (Given)

Therefore, by Basic Proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad ...(i)$$

Again, in ΔACD, we have

$LN \parallel CD$ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \qquad ...(ii)$$

Now, from (i) and (ii), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

5. In ΔPOQ, we have

DE || OQ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \qquad ...(i)$$

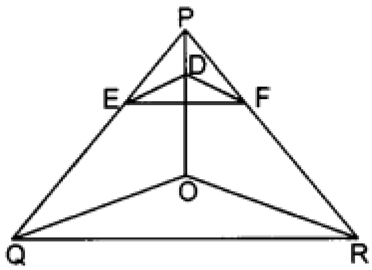


Fig. 7.14

Similarly, in ΔPOR, we have

$DF \parallel OR$ (Given)

$$\frac{PD}{DO} = \frac{PF}{FR} \qquad ...(ii)$$

Now, from (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow \quad EF \parallel QR$$

[Applying the converse of Basic Proportionality Theorem in ΔPQR]

6.

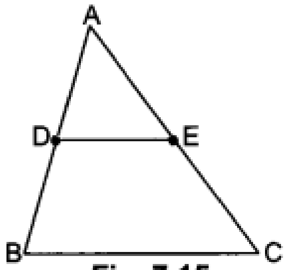


Fig. 7.15

Given: ΔABC in which D and E are the mid-points of sides AB and AC respectively.

To prove: DE || BC

Proof: Since D and E are the mid-points of AB and AC respectively

∴ AD = DB and AE = EC

$$\Rightarrow \quad \frac{AD}{DB} = 1 \quad \text{and} \quad \frac{AE}{EC} = 1$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

DB EC Therefore, DE || BC (By the converse of Basic Proportionality Theorem)

7. (i) In ΔABC and ΔQRP, we have

$$\frac{BC}{RP} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{QP} = \frac{BC}{RP}$

∴ $\Delta ABC \sim \Delta QRP$, by SSS criterion of similarity.

(*ii*) In ΔLMP and ΔFED, we have

$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}, \quad \frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}, \quad \frac{LM}{FE} = \frac{2.7}{5}$$

Hence, $\frac{LP}{FD} = \frac{MP}{ED} \neq \frac{LM}{FE}$

∴ ΔLMP is not similar to ΔFED.

(*iii*) In ΔNML and ΔPQR, we have

$$\angle M = \angle Q = 70°$$

Now, $\frac{MN}{QP} = \frac{2.5}{6} = \frac{5}{12}$ and $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence, $\frac{MN}{QP} \neq \frac{ML}{QR}$

ΔNML is not similar to ΔPQR.

8.

In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD}$ [Given]

So, by *SAS* criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC} \quad \Rightarrow \quad \frac{1}{2} = \frac{5}{DC}$ [$\because AB = 5$ cm]

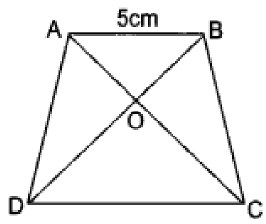
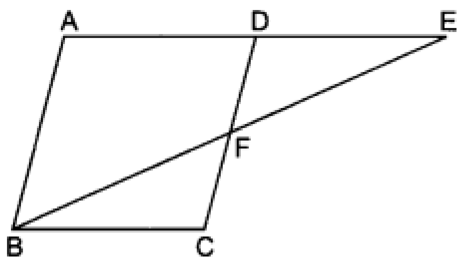


Fig. 7.17

⇒ DC = 10 cm.

9.



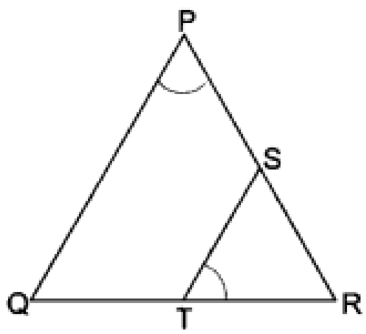
In ΔABE and ΔCFB, we have

∠AEB = ∠CBF (Alternate angles)

∠A = ∠C (Opposite angles of a parallelogram)

∴ ΔABE ~ ΔCFB (By AA criterion of similarity)

10.



In ΔRPQ and ΔRTS, we have

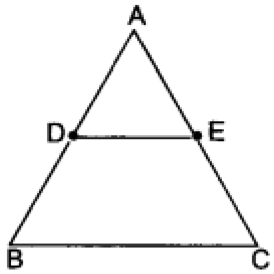
∠RPQ = ∠RTS (Given)

∠PRQ = ∠TRS = ∠R (Common)

∴ ΔRPQ ~ ΔRTS (By AA criterion of similarity)

Long Answer :

1.



Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: AE = EC

Proof: In ΔABC, DE || BC

∴ By Basic Proportionality Theorem, we have

$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AAND}}{\mathrm{ANDC}}$ ... (i)

Now, since D is the mid-point of AB

⇒ AD = BD ... (ii)

From (i) and (ii), we have

$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AAND}}{\mathrm{ANDC}}$

$\Rightarrow 1 = \frac{AAND}{ANDC}$

Hence, E is the mid-point of AC.

2. Given: ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw $OE \parallel AB$ *i.e.*, $OE \parallel DC$.

Proof: In ΔADC, we have $OE \parallel DC$ (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \quad \ldots(i)$$

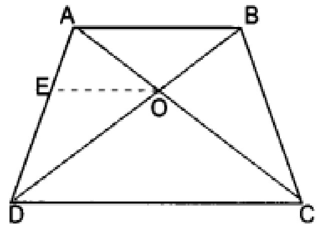


Fig. 7.35

Now, in ΔABD, we have $OE \parallel AB$ (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad \ldots(ii)$$

From (i) and (ii), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

3. In ΔABD and ΔPQM we have

∠B = ∠Q (∵ ΔABC ~ ΔPQR) … (i)

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad (\because \Delta ABC \sim \Delta PQR)$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad \ldots(ii)$$

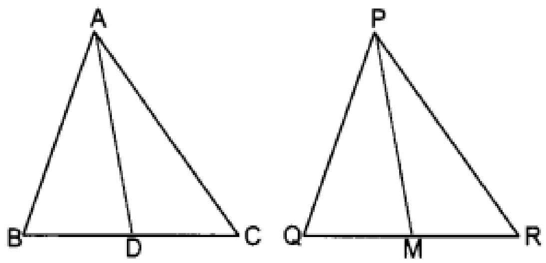


Fig. 7.36

[Since AD and PM are the medians of ΔABC and ΔPQR respectively]

From (i) and (ii), it is proved that

$\Delta ABD \sim \Delta PQM$ (By SAS criterion of similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

4. In ΔEDC and ΔEBA we have

∠1 = ∠2 [Alternate angles]

∠3 = ∠4 [Alternate angles]

∠CED = ∠AEB [Vertically opposite angles]

∴ ΔEDC ~ ΔEBA [By AA criterion of similarity]

$$\Rightarrow \quad \frac{ED}{EB} = \frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC} = \frac{EB}{EA} \quad \ldots(i)$$

It is given that $\Delta AED \sim \Delta BEC$

$$\therefore \quad \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \quad \ldots(ii)$$

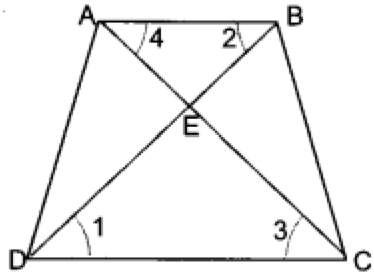


Fig. 7.37

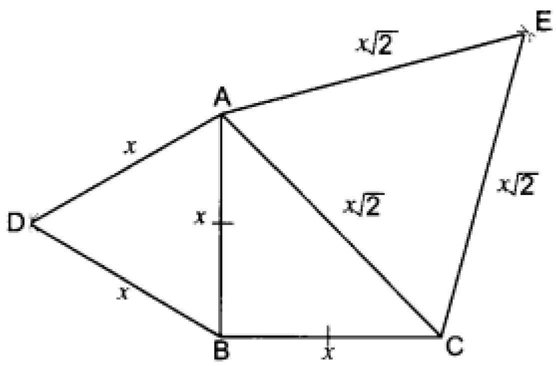
From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \quad \Rightarrow \quad (EB)^2 = (EA)^2 \quad \Rightarrow \quad EB = EA$$

Substituting $EB = EA$ in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC} = 1 \quad \Rightarrow \quad AD = BC$$

5.



Given: A ΔABC in which ∠ABC = 90° and AB = BC.

ΔABD and ΔCAE are equilateral triangles.

To Prove: ar(ΔABD) = $\frac{1}{2}$ × ar(ΔCAE)

Proof: Let AB = BC = x units.

∴ hyp. CA = √x^2 + √x^2 = x√2 units.

Each of the ABD and ΔCAE being equilateral has each angle equal to 60°.

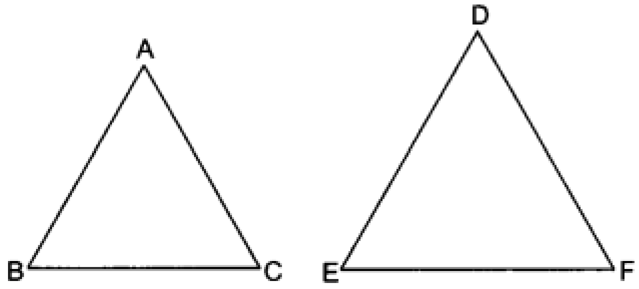
∴ ΔABD ~ ΔCAE

But, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABD)}{ar(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, $ar(\Delta ABD) = \frac{1}{2} \times ar(\Delta CAE)$

6.



Given: Two triangles ABC and DEF, such that

ΔABC ~ ΔDEF and area (ΔABC) = area (ΔDEF)

To prove: ΔABC ≅ ΔDEF

Proof: ΔABC ~ ΔDEF

⇒ ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Now, $ar(\Delta ABC) = ar(\Delta DEF)$ (Given)

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$

and $\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)}$ $(\because \Delta ABC \sim \Delta DEF)$

From (i) and (ii), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \quad \Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

AB = DE, BC = EF, AC = DF

ΔABC ≅ ΔDEF (By SSS criterion of congruency)

7. Let ΔABC and ΔPQR be two similar triangles. AD and PM are the medians of ΔABC and ΔPQR respectively.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \ldots(i)$$

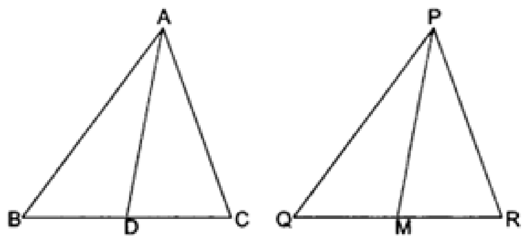


Fig. 7.40

In ΔABD and ΔPQM

$$\frac{AB}{PQ} = \frac{BD}{QM} \qquad \left(\because \frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}\right)$$

and $\angle B = \angle Q$ $\quad$ $(\because \Delta ABC \sim \Delta PQR)$

Hence, $\Delta ABD \sim \Delta PQM$ $\quad$ (By SAS similarity criterion)

$$\frac{AB}{PQ} = \frac{AD}{PM} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

8.

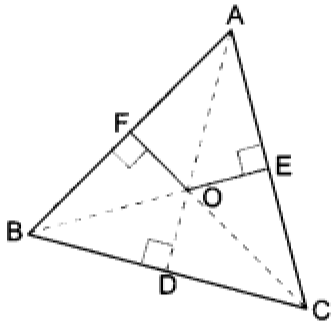


Fig. 7.41

Join OA, OB and OC.

(i) In right Δ's OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$ ... (i)

WHETHER2 = BD^2 + FROM^2 ... (ii)

and $C^2 = EC^2 + OE^2$

Adding (i), (ii) and (iii), we have

⇒ $0A^2 + OB^2 + OC^2 = AF^2 + BD^2 + EC^2 + OF^2 +$ FROM$^2 + OE^2$

⇒ $0A^2 + OB^2 + OC^2 – IP^2 – OE^2 – OF^2 = AF^2 + BD^2 + EC^2$

(ii) We have, $OA^2 + OB^2 + OC^2$ – IP2 – $OE^2 – OF^2$ = AF2 + $BD^2 + EC^2$

⇒ $(OA^2 – OE^2) + (OB^2 – OF^2) – (OC^2 – IP^2) = AF^2 + BD^2 + EC^2$

⇒ $AE^2 + CD^2 + BF^2 = AP^2 + BD^2 + EC^2$

[Using Pythagoras Theorem in ΔAOE, ΔBOF and ΔCOD]

9.

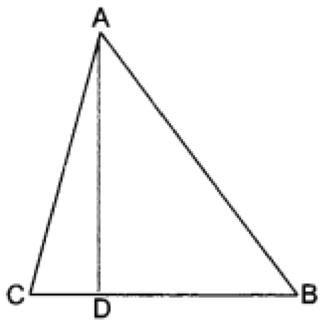


Fig. 7.42

We have, DB = 3CD

Now,

BC = BD + CD

⇒ BC = 3CD + CD = 4CD (Given DB = 3CD)

∴ CD = $\frac{1}{4}$ BC

and DB = 3CD = $\frac{1}{4}$BC

Now, in right-angled triangle ABD using Pythagoras Theorem we have

$AB^2 = AD^2 + DB^2$ ... (i)

Again, in right-angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ ... (ii)

Subtracting (ii) from (i), we have

$AB^2 – AM^2 = DB^2 – CD^2$

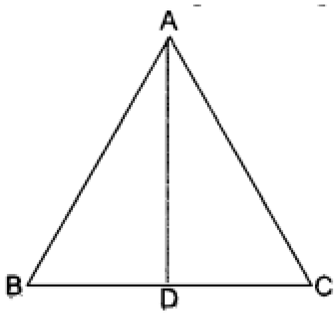
$$\Rightarrow \qquad AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2$$

$$\Rightarrow \qquad AB^2 - AC^2 = \frac{1}{2}BC^2$$

∴ $2AB^2 – 2AM^2 = BC^2$

⇒ $2AB^2 = 2AM^2 + BC^2$

10.



Let ABC be an equilateral triangle and let AD ⊥ BC.

∴ BD = DC

Now, in right-angled triangle ADB, we have

$AB^2 = AD^2 + BD^2$ [Using Pythagoras Theorem]

$$\Rightarrow \quad AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 \quad \Rightarrow \quad AB^2 = AD^2 + \frac{1}{4}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{AB^2}{4} \qquad [\because AB = BC]$$

$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} = AD^2 \quad \Rightarrow \quad \frac{3AB^2}{4} = AD^2 \quad \Rightarrow \quad 3AB^2 = 4AD^2$$

Case Study Answers:

1. Answer :

i	c	90º
ii	b	SAS
iii	b	4 : 9
iv	d	The converse of Pythagoras theorem
v	a	48 cm^2

2. Answer :

i	b	AB, AC
ii	d	$AB^2 = BC^2 + CA^2$
iii	c	6.5m
iv	b	10m
v	a	6.7m (approx)

Assertion Reason Answer-

1. (d) Assertion (A) is false but reason (R) is true
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).